

1 Body

1.1 Definition

For strictly increasing $p_{x,\sigma}$ without skipping,

$$p_{x,\sigma} \equiv \begin{cases} 1, & \text{if } x = 0 \\ -1 \pmod{\sigma}, & \text{if } x \text{ is odd} \\ 1 \pmod{\sigma}, & \text{if } x \text{ is even} \end{cases} \quad (1)$$

and

$$p_{-x,\sigma} = -p_{x-1,\sigma} \quad (2)$$

with

$$p_{a,\sigma} \neq p_{b,\sigma} p_{c,\sigma} \text{ for any } a, b, c. \quad (3)$$

1.2 Remark

$p_{x-2,6}$ gives the x^{th} prime number for $x \geq 4$.

2 Conjecture

Every integer can be represented as a sum of at most $\lfloor \frac{\sigma}{2} \rfloor$ numbers of the form $p_{x,\sigma}$.

3 Appendix

Rough formula for $q_{x,\sigma} = p_{x-1,\sigma}$ for $x \geq \lceil \frac{\sigma+1}{2} \rceil - 1$:

$$q_{x,\sigma} = \sigma + 1 + \sum_{n=\sigma+1}^{2\lfloor x \ln x \rfloor + 2} \left[\frac{2x}{x + \lceil \frac{\sigma+1}{2} \rceil + \sum_{j=1}^{\lfloor \frac{n-1}{\sigma} \rfloor} \left[\frac{-\sum_{k=1}^{\lfloor \frac{\sqrt{\sigma j+1}}{\sigma} \rfloor + 1} \lfloor \text{frac}(\frac{\sigma j+1}{\sigma k+1}) \rfloor - \sum_{k=1}^{\lfloor \frac{\sqrt{\sigma j+1}}{\sigma} \rfloor + 1} \lfloor \text{frac}(\frac{\sigma j+1}{\sigma k-1}) \rfloor}{2 \left(\lfloor \frac{\sqrt{\sigma j+1}}{\sigma} \rfloor + 1 \right)} \right] + \sum_{j=1}^{\lfloor \frac{n+1}{\sigma} \rfloor} \left[\frac{-\sum_{k=1}^{\lfloor \frac{\sqrt{\sigma j-1}}{\sigma} \rfloor + 1} \lfloor \text{frac}(\frac{\sigma j-1}{\sigma k+1}) \rfloor - \sum_{k=1}^{\lfloor \frac{\sqrt{\sigma j-1}}{\sigma} \rfloor + 1} \lfloor \text{frac}(\frac{\sigma j-1}{\sigma k-1}) \rfloor}{2 \left(\lfloor \frac{\sqrt{\sigma j-1}}{\sigma} \rfloor + 1 \right)} \right]} \right] \quad (4)$$

Flatlines at $\sigma + 1$. For $x > 5$ simply add $\lfloor \frac{\sigma}{2} - 2 \rfloor$ to x to complete the formula.