1 Body

1.1 Definition

For strictly increasing $p_{x,\sigma}$ without skipping,

$$p_{x,\sigma} \equiv \begin{cases} 1, & \text{if } x = 0 \\ -1 \mod \sigma, & \text{if } x \text{ is odd} \\ 1 \mod \sigma, & \text{if } x \text{ is even} \end{cases} \tag{1}$$

and

$$p_{-x,\sigma} = -p_{x-1,\sigma} \tag{2}$$

with

$$p_{a,\sigma} \neq p_{b,\sigma} p_{c,\sigma} \text{ for any } a, b, c. \tag{3}$$

1.2 Remark

$p_{x-2,6}$ gives the x^{th} prime number for $x \geq 4$.

2 Conjecture

Every integer can be represented as a sum of at most $\lfloor \frac{\sigma}{2} \rfloor$ numbers of the form $p_{x,\sigma}$.

3 Appendix

Rough formula for $q_{x,\sigma} = p_{x-1,\sigma}$ for $x \geq \lceil \frac{\sigma+1}{2} \rceil - 1$:

$$q_{x,\sigma} = \sigma + 1 + \sum_{n=\sigma+1}^{2\lfloor x \ln x \rfloor + 2} \left\lfloor \frac{2x}{x + \lceil \frac{\sigma+1}{2} \rceil + \sum_{j=1}^{\lfloor \frac{n-1}{\sigma} \rfloor} \left\lfloor \frac{-\sum_{k=1}^{\left\lfloor \frac{\lfloor \sqrt{\sigma j+1} \rfloor}{\sigma} \right\rfloor + 1} \left\lfloor -\operatorname{frac}\left(\frac{\sigma j+1}{\sigma k+1}\right) \right\rfloor - \sum_{k=1}^{\left\lfloor \frac{\lfloor \sqrt{\sigma j+1} \rfloor}{\sigma} \right\rfloor + 1} \left\lfloor -\operatorname{frac}\left(\frac{\sigma j+1}{\sigma k-1}\right) \right\rfloor}{2\left(\left\lfloor \frac{\lfloor \sqrt{\sigma j+1} \rfloor}{\sigma} \right\rfloor + 1\right)} \right\rfloor + \sum_{j=1}^{\lfloor \frac{n+1}{\sigma} \rfloor} \left\lfloor \frac{-\sum_{k=1}^{\left\lfloor \frac{\lfloor \sqrt{\sigma j-1} \rfloor}{\sigma} \right\rfloor + 1} \left\lfloor -\operatorname{frac}\left(\frac{\sigma j-1}{\sigma k+1}\right) \right\rfloor - \sum_{k=1}^{\left\lfloor \frac{\lfloor \sqrt{\sigma j-1} \rfloor}{\sigma} \right\rfloor + 1} \left\lfloor -\operatorname{frac}\left(\frac{\sigma j-1}{\sigma k-1}\right) \right\rfloor}{2\left(\left\lfloor \frac{\lfloor \sqrt{\sigma j-1} \rfloor}{\sigma} \right\rfloor + 1\right)} \right\rfloor} \right\rfloor \tag{4}$$

Flatlines at $\sigma + 1$. For $x > 5$ simply add $\lfloor \frac{\sigma}{2} - 2 \rfloor$ to x to complete the formula.